MATH 54 - HINTS TO HOMEWORK 4

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Here are a couple of hints to Homework 4! Enjoy! :)

SECTION 3.1: INTRODUCTION TO DETERMINANTS

Always try to look for a row/column full of zeros! May Bomberman be with you :)

3.1.19, 3.1.20, 3.1.21, 3.1.22. What you're asked to do is: compute the determinants of the first matrix and of the second matrix and compare them. Also, explain how to obtain the second matrix from the first using a row-operation!

3.1.41. The area of the parallelogram always equals to the determinant of $[\mathbf{u} \quad \mathbf{v}]$. Use the formula: Area of parallelogram $=$ base $\times$ height.

SECTION 3.2: PROPERTIES OF DETERMINANTS

3.2.11. I think what they mean is: First calculate the determinant by expanding along the second column, and then evaluate the resulting sub-determinants using row-reduction!

3.2.19. DO NOT EVALUATE THE DETERMINANT! Use row-reduction! In particular, notice that to obtain the determinant in the problem, all you have to do is multiply the second row of the original matrix by 2, and then add the first row to the second row! Hence, the answer should be $2 \times 7 = 14$.

3.2.21. A matrix A is invertible if and only if $det(A) \neq 0$.

Note: From now on, I'm only giving the answer to the T/F questions! I leave it up to you to explain why the result is true or false.

3.2.27.

(a) **T** ? I think the book uses the term 'row-replacement' to mean: " add k times a row to another row".
(b) **F** (not true for *any* echelon form, what about the reduced row-echelon form?)
(c) **T**
(d) **F**

3.2.31, 3.2.33, 3.2.34, 3.2.35. All you need to use is the fact that $det(AB) = det(A)det(B)$.

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SECTION 3.3: CRAMER'S RULE, VOLUME, AND LINEAR TRANSFORMATIONS

For all those problems, all you need to do is imitate the techniques presented in the book! This section is unlikely to be on the exam (except maybe the volume-part).

3.3.7, 3.3.9. The system has a unique solution iff $det(A) \neq 0$ (because that's equivalent to saying that A is invertible)

3.3.21. The only thing that makes this difficult is that the parallelogram is not centered at $(0, 0)$. To make it centered at $(0, 0)$, just shift it to the right by one unit! The area stays the same anyway!

3.3.25. If $det(A) = 0$, then the columns of A are linearly dependent, and hence A represents a 'degenerate' (or flat) parallepiped, whose volume is 0.

3.3.32. This is similar to the ball-example I showed in section, except that it's even easier, because the problem gives you $T(\mathbf{e_1})$ etc. Also, notice that the volume of S is $\frac{1}{3} \times \frac{1}{2}(1 \times 1) \times 1 = \frac{1}{6}$, because all of its three lengths are equal to 1.

SECTION 4.1: VECTOR SPACES AND SUBSPACES

Remember the three techniques of showing whether something is a vector space or not!

(1) Trick 1: Show it is not a vector space by finding an explicit property which does not hold
(2) Trick 2: Show it is a subspace of a (known) vector space
(3) Trick 3: Express it in the form $Span$ of some vectors.

4.1.24.

(a) **T** (this is important to remember!!! A vector isn't a list of numbers any more, it could be anything, even a function!)
(b) **T**
(c) **T** (of itself!)
(d) **F**
(e) **T** (again, the textbook might give you a different answer, but I agree that this is weirdly phrased! What they mean is: If $\mathbf{u}, \mathbf{v}$ is in H, then $\mathbf{u} + \mathbf{v}$ is in H).

4.1.28.

(a) 8
(b) 3
(c) 5
(d) 4

4.1.32. This is a bit tricky! Remember that $H \cap K$ is the set of vectors that is both in H and in K. Here's the proof that $H \cap K$ is closed under addition (hopefully that'll inspire you to do the rest):

Suppose $\mathbf{u}$ and $\mathbf{v}$ are in $H \cap K$. Then $\mathbf{u}$ and $\mathbf{v}$ are in H, so is $\mathbf{u} + \mathbf{v}$ (since H is a subspace). Also, since $\mathbf{u}$ and $\mathbf{v}$ are in K, so is $\mathbf{u} + \mathbf{v}$ (since K is a subspace). Hence $\mathbf{u} + \mathbf{v}$ is both in H and K, hence $\mathbf{u} + \mathbf{v}$ is in $H \cap K$.

As for the fact that the union of two subspaces is not a subspace, take H to be the $x-$axis, and K to be the $y-$ axis.

SECTION 4.2: NULLSPACES, COLUMN SPACES, AND LINEAR TRANSFORMATIONS

This is very similar to what you've been doing in sections 2.8 and 2.9.

4.2.25.

(a) **T**
(b) **F**
(c) **T**
(d) **T** (the book might say **F**, if it is pedantic about the fact that it didn't say 'for all $\mathbf{b}$')
(e) **T**
(f) **T**

SECTION 4.3: LINEARLY INDEPENDENT SETS, BASES

Remember that a basis is a linearly independent set which spans the whole space! Equivalently, as set is a basis if the corresponding matrix A is invertible.

4.3.11. The system the book talks about is:

$$\begin{cases} x + 2y + z = 0 \\ y = y \\ z = z \end{cases}$$

4.3.17. **IGNORE THIS PROBLEM**, unless you enjoy being tortured :)

4.3.21.

(a) **F**
(b) **F**
(c) **T**
(d) **F**
(e) **F**

4.3.32. This is the same as problem 3 in Quiz 2!

4.3.33. Suppose $a\mathbf{p}_1 + b\mathbf{p}_2 = \mathbf{0}$, where $\mathbf{0}$ is the zero-polynomial. Then either identify coefficients (like in Math 1B), or differentiate, to find out that $a = b = 0$.